

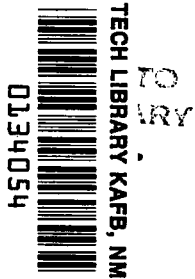
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STRATEGIES FOR ESTIMATING THE MARINE GEOID FROM ALTIMETER DATA

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16. Abstract In processing altimeter data from a spacecraft-borne altimeter to estimate the fine structure of the marine geoid, a problem is encountered. To describe the geoid fine structure, a large number of parameters must be employed, and to estimate all of them simultaneously is not possible. In practice, one is forced to hold a large number of parameters at <i>a priori</i> values and adjust others. Unless the parameterization exhibits good orthogonality in the data, serious aliasing results. Simulation studies show that, among several competing parameterizations, the mean free-air gravity anomaly model (i.e., Stokes' formula) exhibited promising geoid recovery characteristics. Using covariance analysis techniques, this document provides quantitative measures of the orthogonality properties associated with the previously mentioned parameterization. For instance, a 5- by 5-degree area mean free-air gravity anomaly can be estimated with an uncertainty of 0.01 mm/s ² (1 mgal) (40-cm undulation) if all free-air gravity anomalies within a spherical radius of 10 arc degrees are simultaneously estimated.					
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STRATEGIES FOR ESTIMATING THE MARINE GEOID FROM ALTIMETER DATA

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INTRODUCTION

The primary function of the spacecraft-borne altimeter is to determine the fine structure of the mean sea-surface topography. The instrument is well suited for the task. Consider, for instance, the altimeter on board the Geodetic Earth-Orbiting Satellite-C (GEOS-C) spacecraft which was launched in late 1974. In its global mode, the instrument is capable of producing a measurement every 4 kilometers along the subearth path of the satellite. This implies that, even assuming considerable data compression, it will be mathematically possible to extract topographic detail of 1 degree or less from such data.

But practical difficulties must be overcome before the full potential of altimetry as a data type can be realized. To determine these difficulties, standard estimation techniques used to obtain sea-surface topography from altimetry data must be closely analyzed. Essentially, the problem is to reconstruct an analytic surface from direct but noisy observations of the surface taken at specified points. The obvious approach is to parameterize the surface in terms of coefficients which define a suitably dense set of functions in the space of two dimensional analytic functions and to recover the coefficients from the data by means of a standard minimum variance filter. The physical setting of the problem often suggests the proper parameterization. If not, an arbitrary family of analytic functions such as the set of two dimensional polynomials can be used. For this problem, a natural parameterization is suggested by the fact that, after suitable corrections, the mean sea surface can be considered as cohering closely to an equipotential surface known as the marine geoid (Reference 1). Thus, one candidate for a natural parameterization is the set of standard spherical harmonic coefficients of the Earth's geopotential field.

Any parameterization capable of describing the fine structure of the surface in question must employ an enormous number of coefficients. For instance, determination of 3-degree features of the marine geoid using the standard spherical harmonic expansion of the geopotential field as a parameterization would require a full set of coefficients up to degree and order 60. This implies the estimation of over 3700 parameters. Unless special circumstances apply, it is impossible to simultaneously estimate such large parameter sets. In practice,

to use parameter estimation techniques in recovering the marine-geoid fine structure from altimeter data, it will be necessary to adjust small subsets of the parameters while "freezing" the rest at *a priori* values. But, unless the parameterization exhibits a certain property with regard to altimetry data which is termed "orthogonality in the data set," uncertainties in the adjusted terms will badly corrupt the estimates of the adjusted terms. This effect is frequently called aliasing. The orthogonality property just mentioned (defined in detail in a later section) is a property of a parameterization which permits a decomposition of the estimation problem into estimation problems of much smaller dimensionality without fear of serious aliasing.

To take full advantage of the attractive properties of altimetry, it is necessary to develop a parameterization of the marine geoid which exhibits good orthogonality properties in altimeter data. The orthogonality properties of the set of spherical harmonic coefficients of the geopotential field are poor and are not good candidates for the proper parameterization. Other candidates for the parameterization of the marine geoid are surface density layers (Reference 2) and sample functions (Reference 3). The parameterization whose properties are investigated in this paper is the one provided by mean free-air gravity anomalies and Stokes' formula (References 4 and 5). If this parameterization is used to determine the marine geoid from altimeter data, it is important to determine to what extent mean free-air gravity anomalies are orthogonal in the data. Specifically, the important factor is how far two mean free-air gravity anomalies must be separated to ensure that neglecting one anomaly does not badly alias the estimate of the other. Without knowledge of this distance, intelligent use of the parameterization is impossible.

Results of the Skylab altimeter experiment have demonstrated the ability of altimetry to reveal marine-geoid fine structure. However, the GEOS-C mission will provide the first opportunity for scientists to examine the output of a spacecraft-borne altimeter functioning in a global mode over the oceans of the world. The next section details the performance of the Skylab altimeter and the goals of the GEOS-C experiment. The mathematical details of the measurement modeling and preprocessing of altimeter data will be described in a later section, and the Stokes' formula mean free-air gravity anomaly parameterization of the marine geoid will be documented. Following that will be a detailed discussion of the dual concepts of orthogonality and aliasing and their relationships to the problem of estimating a marine geoid from altimeter data. Finally, the results of a systematic application of covariance analysis will be used to develop optimal estimation strategies for determining the marine geoid from altimetry data.

SKYLAB AND GEOS-C ALTIMETER EXPERIMENTS

The GEOS-C spacecraft was inserted into orbit during the latter part of calendar year 1974. The prime experiment of this spacecraft was a radar altimeter. The objectives of the GEOS-C altimeter experiment were: (a) to determine the feasibility and utility of a spaceborne altimeter to map the topography of the ocean surface with an absolute height accuracy of ± 5 meters and with a relative height accuracy of 1 to 2 meters; (b) to determine the feasibility of measuring the deflections of the vertical at sea; (c) to determine

the feasibility of measuring wave height; and (d) to contribute to technology leading to a future operational satellite-altimeter system with a 10-cm measurement capability.

When calibrated and corrected (e.g., for sea state, ocean tides, and other effects), the altimeter data constitute measures of the distance between the GEOS-C spacecraft and the ocean surface. Knowledge of the satellite altitude relative to a reference ellipsoid and knowledge of the oceanographic departures of the sea-surface topography from the geoid will then permit the determination of the geoid. The chief problem expected is the determination of orbital altitude for GEOS-C. The primary tracking systems for doing this are the satellite-to-satellite tracking system and precision lasers. Data from these and other systems tracking GEOS-C, including C-band radars, USB range and range-rate stations, and TRANET Doppler stations, will be used to find satellite height.

Satellite contributions to the determination of the current ocean geoid have spatial resolutions corresponding to half wavelengths of approximately 18 degrees (2000 km). Although surface gravity achieves representations with finer resolution, in the 1- to 5-degree (110- to 550-km) range, it covers only about 50 percent of the ocean surface. The GEOS-C altimeter system will therefore fill in the gaps and provide valuable independent knowledge where data now exist.

The Skylab altimeter has recently demonstrated the ability of a radar altimeter to detect features of the ocean surface. Figure 1 is an analysis of an altimeter pass over the Puerto Rican Trench. The pass was over the southwest corner of Puerto Rico and was 72.8 seconds in duration. A plot of the height residuals (formed by comparing the altimeter measurements with the calculated height of Skylab's orbit) shows that a 17-meter drop in height residuals occurred when Skylab passed over the deepest part of the ocean (i.e., corresponding to a 4000-meter depth of the ocean bottom) and that the peaking of the height residuals occurred when Skylab traversed the Puerto Rican land mass. The Skylab data described here exemplify the high level of resolution of surface features by a radar altimeter.

Note, however, that data from the GEOS-C experiment will be significantly free of effects of spacecraft dynamic motions, (which is not the case for Skylab altimeter data because of attitude-control jet thrusting, crew motion, etc.). In addition, because GEOS-C had a nominal 1-year operating lifetime, the altimetry data obtained was far greater than that obtained from Skylab. For reasons previously cited, the GEOS-C altimeter experiment data are more suitable for improving the marine geoid.

To successfully use GEOS-C altimetry data for improving the accuracy of the marine geoid, the development of unique computer programs capable of processing these data was necessary. The following sections describe the mathematical models associated with these programs and the computation strategies for processing altimeter data.

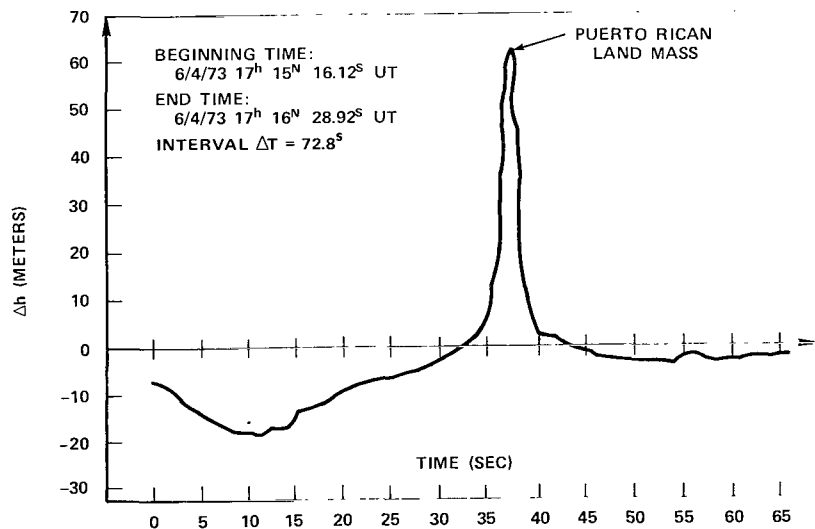
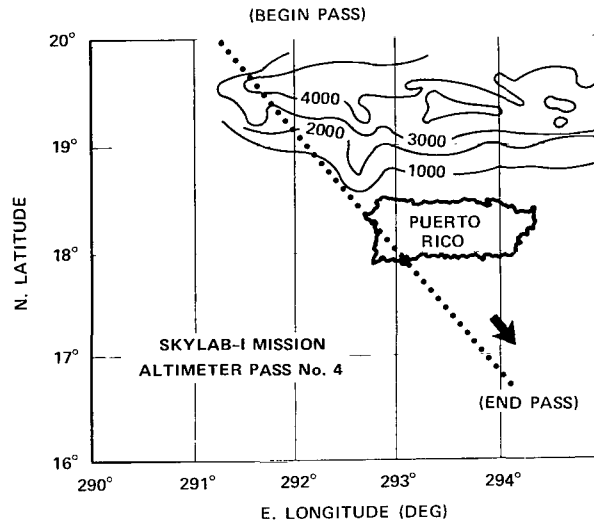


Figure 1. Altimeter pass over the Puerto Rico Trench: (a) ground track of altimeter pass; (b) Skylab-I altimeter residuals versus time data source: NASA/Wallops Station.

DESCRIPTION OF ALTIMETER MEASUREMENT

Figure 2 shows the geometry associated with the altimeter measurement. As can be seen, the altimeter measurement is nominally the shortest distance between the satellite and the sea surface (i.e., the measurement is along the normal to the sea surface that passes through

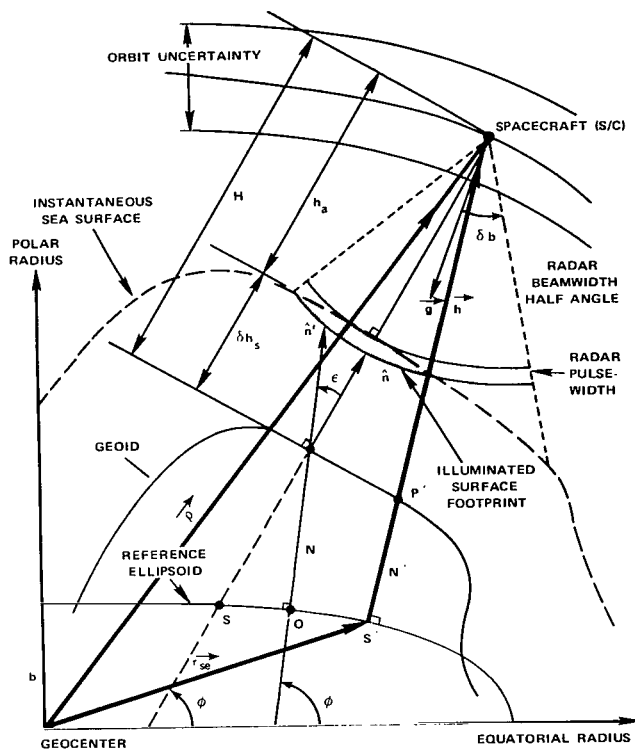


Figure 2. Altimeter measurement geometry.

the satellite). The following relationship gives the mathematical model for the altimeter measurement:

$$h_a = h - N' - \delta h_s - \Delta h' \quad (1)$$

where

$$h = |\vec{\rho} - \vec{r}_{se}|$$

$$\vec{\rho} = \text{position vector of S/C}$$

$$\vec{r}_{se} = \text{geocentric radius vector}$$

$$N' = \text{geoid height above reference ellipsoid}$$

$$\delta h_s = \text{deviation of sea surface from geoid}$$

$$\Delta h = \text{systematic errors in altimeter measurement (e.g., refraction, timing, etc.)}$$

$$\begin{aligned} \vec{\rho} &= \vec{r}_{se} + \vec{h} \\ \vec{h} &= \vec{\rho} - \vec{r}_{se} \\ h &= |\vec{h}| = N' + h_a + \delta h_s + \Delta h \\ h_a &= h - N' - \delta h_s - \Delta h' \end{aligned}$$

WHERE:

- $\vec{\rho}$: POSITION VECTOR OF S/C
- \vec{n} : VECTOR FROM S ON REFERENCE ELLIPSOID TO S/C
- \vec{r}_{se} : GEOCENTRIC RADIUS VECTOR TO POINT S ON REFERENCE ELLIPSOID.
- h : MAGNITUDE OF \vec{h}
- h_a : S/C ALTITUDE ABOVE SEA SURFACE
- N' : GEOID HEIGHT ABOVE REFERENCE ELLIPSOID
- δh_s : DEVIATION OF SEA SURFACE FROM GEOID
- Δh : SYSTEMATIC ERRORS IN ALTIMETER MEASUREMENT, e.g., REFRACTION, ANTENNA OFFSET, TIMING, ETC.

A more detailed description of the mathematical modeling of the altimeter measurement appears in Reference 6.

The error sources that effect the altimeter measurement fall into three categories: (1) those that result from orbit uncertainty; (2) those that cause the measured geoid to deviate from the true geoid; and (3) those that affect the measurement accuracy itself. Equation 2 describes the functional dependence of the error sources on the altimeter measurement:

$$h_a = h(\vec{E}_0 t) - N'(\vec{G}_0) - \delta h_s(\vec{\tau}_0, \vec{s}_0, \phi, \lambda, t) - \Delta h'(\vec{m}_0, \phi, \lambda, t) \quad (2)$$

where

- h = S/C altitude above reference ellipsoid
- E = orbit parameters and orbit dependent terms (radiation pressure, drag, etc.)
- t = time of altimeter measurement
- \vec{G}_0 = vector of geopotential coefficients
- N' = geoid undulation function
- $\vec{\tau}_0$ = vector of tidal error model coefficients
- \vec{s}_0 = vector of local sea-surface biases (i.e., currents, storm surges, wind waves . . .)
- \vec{m}_0 = vector of altimeter measurement error coefficients

or in general

$$h_a = h_a(\vec{E}_0, \vec{G}_0, \vec{\tau}_0, \vec{s}_0, \vec{m}_0, \phi, \lambda, t) \quad (3)$$

The altimeter measurement will be corrected for known error sources, smoothed, sorted, etc., by the data preprocessor. Each measurement will then be compared with altitude calculated from the satellite's orbit (h_a') to form the measurement's residual Δh_a

$$\Delta h_a = h_a - h_a' \quad (4)$$

Mathematically, this residual is equivalent to the difference function,

$$\begin{aligned} \Delta h_a = & \frac{\partial h_a}{\partial E_i} \Delta E^i + \frac{\partial h_a}{\partial G_j} \Delta G^j + \frac{\partial h_a}{\partial \tau_k} \Delta \tau^k \\ & + \frac{\partial h_a}{\partial s_\ell} \Delta s^\ell + \frac{\partial h_a}{\partial m_r} \Delta m^r + \epsilon \end{aligned} \quad (5)$$

or

$$\Delta h_a = \Delta h_a(E) + \Delta h_a(G) + \Delta h_a(\tau) + \Delta h_a(s) + \Delta h_a(m) \quad (5a)$$

Models for the altimeter measurement errors follow.

Error Attributable to Orbital Parameters ($\Delta h_a(E)$)

Precision laser tracking, satellite-to-satellite tracking (SST), USB, C-band, and Doppler data preceding and during altimeter measurement data acquisition will be used to correct the error on altimeter measurements attributable to orbital parameters using an existing orbit-determination program. Orbit-height accuracy required for altimetry data reduction must be better than 1 meter. Recent studies indicate that this is feasible (Reference 7).

Error Attributable To Tides And Sea Surface ($\Delta h_a(\tau)$ and $\Delta h_a(S)$)

Corrections for deviations from the geoid attributable to ocean tides are based on the model of Hendershott (Reference 8). This model represents the dominant lunar semidiurnal tide (M_2) with lunar declination terms neglected and is representative of the current state of the art. The maximum error contribution attributable to tides is approximately 1 meter. The contributions attributable to local sea-surface biases are on the order of 50 cm, below the resolution threshold of the GEOS-C altimeter. Therefore, sea-surface bias corrections are neglected.

Errors Attributable To Altimeter Hardware ($\Delta h_a(m)$)

Altimeter measurement errors which directly affect the accuracy for determining the geoid radius are:

$$\Delta h_a(m) = \Delta h_0 + \Delta h_\ell + \Delta h_L + \Delta h_D + \Delta h_T + \Delta h_R \quad (6)$$

where

Δh_0 = altimeter antenna offset

Δh_ℓ = antenna offset due to S/C libration

Δh_L = dynamic lag

Δh_D = altimeter drift

Δh_T = timing bias

Δh_R = tropospheric refraction

Models for the foregoing error sources follow.

Altimeter Antenna Offset

$$\begin{aligned}\Delta h_0 = & h_a \tan \frac{1}{2}(\alpha_0 - \delta_b) \tan(\alpha_0 - \delta_b) \\ & + \left[\frac{(c\tau)^3}{4\delta_b^2 h_a} \right]^{\frac{1}{2}} \left[\left(\frac{\alpha_0}{2\delta_b} \right)^2 - .225 \right]\end{aligned}\quad (6a)$$

for

$$\delta_b \leq \alpha_0 < 2^\circ$$

or

$$\Delta h_0 = \left[\frac{(c\tau)^3}{4\delta_b^2 h_a} \right]^{\frac{1}{2}} \left[\left(\frac{\alpha_0}{2\delta_b} \right)^2 - .225 \right]\quad (6b)$$

for

$$0 < \alpha_0 < \delta_b$$

where

h_a = spacecraft altitude

δ_b = antenna half beamwidth angle

α_0 = angle off boresight

τ = transmitted pulse length

c = velocity of light (299.7925×10^3 km/sec)

Antenna Offset Attributable to Spacecraft Libration

$$\begin{aligned}\Delta h_\ell = & h_a \tan \frac{1}{2}(\alpha_\ell - \delta_b) \tan(\alpha_\ell - \delta_b) \\ & + \left[\frac{(c\tau)^3}{4\delta_b^2 h_a} \right]^{\frac{1}{2}} \left[\left(\frac{\alpha_\ell}{2\delta_b} \right)^2 - .225 \right]\end{aligned}\quad (6c)$$

for

$$\delta_b \leq \alpha_\ell < 2^\circ$$

or

$$\Delta h_{\ell} = \left[\frac{(c\tau)^3}{4\delta_b^2 h_a} \right]^{1/2} \left[\left(\frac{\alpha_{\ell}}{2\delta_b} \right)^2 - .225 \right] \quad (6d)$$

for

$$0 < \alpha_{\ell} < \delta_b$$

where

$$\alpha_{\ell} = A_{\ell} \sin \left(\frac{2\pi}{T} t + \phi_{\ell} \right)$$

α_{ℓ} = S/C libration angle

A_{ℓ} = peak libration angle

ϕ_{ℓ} = libration phase angle

T = orbital period

Dynamic Lag

Dynamic lag is a measurement-bias error induced by the change in spacecraft position at the time of the outgoing and return pulses.

$$\Delta h_L = \frac{\dot{h}_{i+1} - \dot{h}_i}{(t_{i+1} - t_i)} \mathcal{L} \quad (6e)$$

where

\dot{h}_i = altitude rate at time t_i (sec)

\mathcal{L} = lag coefficient (sec²)

Altimeter Drift

Altimeter drift is a measurement-bias error attributable to component aging.

$$\Delta h_D = D(t - t_0) \quad (6f)$$

where

t_0 = initial time of altimeter measurement

t = current time of altimeter measurement

D = drift coefficient (sec)

Timing Bias

Timing bias is bias error introduced by spacecraft clock error.

$$\Delta h_T = \dot{h}\tau \quad (6g)$$

\dot{h} = height rate at time of altimeter measurement

τ = clock error coefficient

Tropospheric Refraction Error

Tropospheric refraction error is a bias error introduced into altimeter measurement because of tropospheric refraction. The model used is that developed by J. Saastamoinen (Reference 9).

Error Attributable to Geopotential $\Delta h_a(G)$

In the preprocessing of altimeter data, the altimeter measurement will have been corrected for the orbit uncertainties of ocean tides, local sea-surface biases, and measurement-bias errors using the error models described previously. Thus equation 4 is restated as follows:

$$\Delta h_a(G) = h_a - [h'_a + \Delta h_a(E) + \Delta h_a(\tau) + \Delta h_a(s) + \Delta h_a(m)] \quad (7)$$

The relationship between altimeter measurement residuals to geoidal parameters can therefore be stated as follows:

$$\Delta h_a(G) \approx \frac{\partial h_a}{\partial G_j} \Delta G_j \quad (8)$$

As stated previously, the altimeter measurement residual as described in Reference 7 solely reflects primarily (neglecting second order effects) the departure of the geoid from the reference ellipsoid or, more precisely, the distance along the unit normal to the reference ellipsoid between the reference ellipsoid and the geoid which is called geoidal height or geoidal undulation N' (figure 2).

The mathematical relationship which relates the geoidal undulation to the disturbing potential T is given by Bruns' formula derived in Reference 4. That is,

$$N = \frac{T}{\gamma} \quad (9)$$

where

T = disturbing potential

γ = magnitude of gravity vector normal to reference ellipsoid

The disturbing potential of the global geoid at point (ϕ, λ, r) is expressed in terms of spherical harmonics as follows:

$$T = \frac{GM}{r} \sum_{n=2}^L \left(\frac{a}{r}\right)^n \sum_{m=0}^n P_{nm}(\sin \phi) [C_{nm} \cos m\lambda + S_{nm} \sin m\lambda] \quad (10)$$

where

GM = gravitational constant of the earth

C_{nm}, S_{nm} = spherical harmonic expansion coefficients

$P_{nm}(\sin \phi)$ = associated Legendre functions

(ϕ, λ) = latitude and longitude on geoid at which disturbing potential is evaluated

r = geocentric radius from earth center of mass to evaluation point of disturbing potential on geoid

a = semimajor axis of reference ellipsoid

L = is the limit of summation, and it is specified by the degree of harmonic expansion of the global geoid

n = summation index for degree terms of the spherical harmonic expansion of T

m = summation index for the order of terms in spherical harmonic expansion of T

Thus, the geoidal undulation at any point P (ϕ, λ, r) on the earth can be computed from geopotential coefficients derived from satellites by analysis of perturbations on the orbits induced by the Earth's gravity field. The undulations are computed from the combination of equations 9 and 10.

The Stokes' formula (Reference 4, pp. 92-98) is another form of expressing the disturbing potential. This formula makes it possible to express the disturbing potential of the global geoid from gravity data. That is,

$$T = \frac{R}{4\pi} \int_{\sigma} \Delta g S(\psi) d\sigma \quad (11)$$

where

- a = radius of reference ellipsoid
- f = flattening of reference ellipsoid
- $S(\psi)$ = Stokes' function
- σ = element of area
- Δg = surface gravity anomalies

$$S(\psi) = \csc\left(\frac{\psi}{2}\right) - 6 \sin \frac{\psi}{2} + 1 - 6 \cos \psi - 3 \cos \psi \ln\left(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2}\right) \quad (11a)$$

from Bruns' formula (equation 9).

The geoidal undulation at any point P on the global geoid can be computed from Stokes' formula (equation 11). That is,

$$N = \frac{T}{\gamma} = \frac{R}{4\pi\gamma} \int_{\sigma} \Delta g S(\psi) d\sigma \quad (11b)$$

In terms of geographical coordinates, Stokes' function can be expressed as follows:

$$N(\phi, \lambda) = \frac{R}{4\pi\gamma} \int_{\lambda'=0}^{2\pi} \int_{\phi'=-\pi/2}^{\pi/2} \Delta g(\phi', \lambda') S(\psi) \cos \phi' d\phi' d\lambda' \quad (11c)$$

where

$$d\sigma = \cos \phi' d\phi' d\lambda'$$

(ϕ, λ) = latitude and longitude of the computation point

(ϕ', λ') = coordinates of the variable surface element σ

ψ = spherical distance between the computation point and variable surface element

$$\psi = \cos^{-1} [\sin \phi \sin \phi' + \cos \phi \cos \phi' \cos (\lambda - \lambda')]$$

$\Delta g(\phi', \lambda')$ = free-air gravity anomaly at the variable point (ϕ', λ')

γ = mean value of gravity over Earth

To combine surface gravity data and geopotential information derived from gravity field perturbations acting on orbits of spacecraft, the Earth is divided into two areas (Reference 10), a global area (A_1) and a local area (A_2), surrounding the point P. Each gravity anomaly in each area is also partitioned into two parts represented by Δg_s and Δg_2 respectively, where

$$\Delta g_s = \gamma \left[\sum_{n=2}^L \sum_{m=0}^n (n-1) \left(\frac{a}{r} \right)^n P_{nm}(\sin \phi) \{C_{nm} \cos n\lambda + S_{nm} \sin m\lambda\} \right] \quad (11d)$$

The Δg_2 value is defined as the remainder of the gravity anomaly. By partitioning the Earth into two areas and the geoidal undulations into two components, equation 11c can therefore be rewritten as follows:

$$N(\phi, \lambda) = N_1 + N_2 \quad (11e)$$

where

$$N_1 = \frac{R}{4\pi\gamma} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \Delta g_s(\phi', \lambda') S(\psi) \cos \phi' d\phi' d\lambda'$$

or

$$= \frac{T}{\gamma}$$

and

$$N_2 = \frac{R}{4\pi\gamma} \int_{A_2} \int \Delta g_2(\phi', \lambda') S(\psi) \cos \phi' d\phi' d\lambda'$$

$$N_2 = \Delta N_1 = \delta \tilde{N} = \frac{\Delta T}{\gamma}$$

$$\Delta g_2 = \delta(\Delta g_s)$$

Thus,

$$N(\phi, \lambda) = \frac{T + \Delta T}{\gamma} \quad (11f)$$

$$N_2 = \Delta N_1 = \frac{R \Delta \phi' \Delta \lambda'}{4\pi\gamma} \sum_{j=1}^{\bar{L}} \delta\{\Delta g_s(\phi'_j \lambda'_j)\} S(\psi_j) \cos \phi'_j$$

where ΔN_1 is the correction to the geoidal undulations of the global geoid as a function of the corrections of mean free-air gravity anomalies.

Equation 11f is the form of the parameterization adapted for relating the altimeter measurement residuals to geoidal parameters. That is, equation 8 can now be written as follows:

$$\begin{aligned} \delta \tilde{N} &= \Delta h_a = \Delta N_1 \\ \delta \tilde{g} &= \Delta G_j = \delta\{\Delta g_s(\phi'_j \lambda'_j)\} \\ A &= \frac{\partial h_a}{\partial G_j} = \frac{R \Delta \phi' \Delta \lambda'}{4\pi\gamma} S(\psi_j) \cos \phi'_j \end{aligned} \quad (12)$$

or in the matrix form

$$\delta \tilde{N}_{(k \times 1)} = A_{(k \times j)} \delta \tilde{g}_{(j \times 1)} \quad (12a)$$

ORTHOGONALITY AND ALIASING

Assuming that altimeter observations can be directly translated into deviations of the marine geoid from a reference geoid, it is straightforward to estimate gravity anomalies from altimetry data. Let $\delta\tilde{N}$ be a vector of geoid undulations collected from a certain region over the ocean. Next, let $\delta\tilde{g}$ be a vector of mean free-air gravity anomalies defined over a region of the ocean which contains the data region. Then, repeating equation 12,

$$\delta\tilde{N} = A \delta\tilde{g} \quad (13)$$

where A is a matrix the number of whose rows is equal to the number of data points and the number of whose columns is equal to the number of gravity anomalies. As demonstrated in equation 12, the individual elements of A (for example, $A(I, J)$) can be computed through Stokes' formula and the latitude and longitude of the i^{th} data point, together with the longitude and latitude of the midpoint of the grid over which the J^{th} gravity anomaly is defined. Equation 13 provides a linear equation of condition, and, in a standard minimum variance fashion, $\delta\tilde{g}$ could be estimated from observations of $\delta\tilde{N}$. However, in order for equation 13 to be correct (correct, that is, assuming that the approximations inherent in the discrete form of Stokes' formula are valid), the gravity anomalies in the array $\delta\tilde{g}$ must cover the globe. Considerably smaller regions would no doubt be adequate, but just how small these regions can be before a serious bias is introduced into the estimation process must be determined by computations. In any case, computational considerations necessitate the choice of a region in which the number of elements in the $\delta\tilde{g}$ array does not exceed two or three hundred. Gravity anomalies outside this region are assumed to be zero. To see precisely what happens when this assumption is made, postulate that the $\delta\tilde{g}$ array of equation 13 is defined over an area so large that equation 13 is substantially correct. Then write

$$\delta\tilde{g} = \begin{bmatrix} \delta\tilde{g}_1 \\ \delta\tilde{g}_2 \end{bmatrix} \quad (14)$$

where

$\delta\tilde{g}_1$ = gravity anomalies to be adjusted in a standard minimum variance filter

and

$\delta\tilde{g}_2$ = gravity anomalies assumed to be zero and thus left unadjusted by the minimum variance filter.

Then equation 13 can be written

$$\delta\tilde{N} = A_1 \delta\tilde{g}_1 + A_2 \delta\tilde{g}_2 \quad (15)$$

where A_1 and A_2 are the variational matrices of $\delta\tilde{N}$ with respect to $\delta\tilde{g}_1$ and with respect to $\delta\tilde{g}_2$, respectively.

After proper corrections, altimeter data are treated as direct observations δN of $\delta\tilde{N}$ with statistics

$$\delta N = \delta\tilde{N} + \nu, E(\nu) = \bar{0}, E(\nu\nu^T) = Q \quad (16)$$

Estimates of mean free-air gravity anomalies obtained from satellite tracking and gravimetry measurements are available (Reference 10). Unless this information was correctly factored into the gravity anomaly estimates obtained from altimeter data, the resultant estimates would not be optimal. Consequently, we assume the existence of an *a priori* estimate $\delta g_1'$ of δg_1 with properties

$$\delta g_1' = \delta\tilde{g}_1 + \alpha_1, E(\alpha_1) = \bar{0}, E(\alpha, \alpha_1^T) = P_1 \quad (17)$$

For computational reasons, the gravity anomalies $\delta\tilde{g}_2$ are assumed to be zero, but actual values of gravity anomalies in the region on the sphere which is ignored have a certain distribution about zero. We assume

$$E(\delta\tilde{g}_2) = \bar{0}, E(\delta\tilde{g}_2 \delta\tilde{g}_2^T) = P_2 \quad (18)$$

When the values of $\delta\tilde{g}_2$ are assumed to be zero, the minimum variance estimate of $\delta\tilde{g}_1$ becomes

$$\delta\hat{g}_1 = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} [A_1^T Q^{-1} \delta N + P_1^{-1} \delta g_1'] \quad (19)$$

See Reference 11 for details. Define the covariance matrix of the estimator given by equation 19 as

$$P = E[(\delta\hat{g}_1 - \delta\tilde{g}_1)(\delta\hat{g}_1 - \delta\tilde{g}_1)^T] \quad (20)$$

From equations 15, 16, 17, and 19, we obtain

$$\delta\hat{g}_1 - \delta\tilde{g}_1 = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} (-A_1^T Q^{-1} A_2 \delta\tilde{g}_2 + A_1^T Q^{-1} \nu + P_1^{-1} \alpha_1) \quad (21)$$

Equation 21 yields

$$P = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} + (A_1^T Q^{-1} A_1 P_1^{-1})^{-1} A_1^T Q^{-1} A_2 P_2 A_2^T Q^{-1} A_1 \\ (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} \quad (22)$$

Assume that the data is uncorrelated and that each data point has the same variance. Then

$$Q = I\sigma_0^2 \quad (23)$$

where I is the identity matrix and σ_0^2 is the common variance of the data. Also assume that the values of the unadjusted gravity anomalies are independently distributed. Then the covariance matrix P_2 of $\delta\tilde{g}_2$ can be written as

$$P_2 = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & 0 & 0 & \\ & & & \sigma_{n_2}^2 \end{bmatrix} \quad (24)$$

where n_2 is the number of unadjusted gravity anomalies and σ_i^2 is the second moment about zero of the i^{th} unadjusted gravity anomaly.

Also define a matrix K as

$$K = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} A_1^T Q^{-1} A_2 \quad (25)$$

If n_1 is the number of adjusted parameters, then K is of dimension n_1 by n_2 . With these assumptions, equation 22 yields the following expression for the variance of the i^{th} adjusted gravity anomaly

$$P(I,I) = \sum_{j=0}^{n_2} (\beta_{i,j} \sigma_j)^2 \quad (26)$$

where $\beta_{i,0}^2$ is the i^{th} diagonal element of matrix $(A_1^T A_1)^{-1}$ (assume here that diagonal elements of matrix P_1^{-1} are relatively small) and

$$\beta_{i,j} = K(I,J), J \geq 1 \quad (27)$$

Define the error-sensitivity matrix as

$$s = \{\beta_{i,J}\}, i = 1, 2, \dots, n_1, J = 0, 1, 2, \dots, n_2 \quad (28)$$

and, finally, define the alias matrix as

$$L = s\bar{\sigma} \quad (29)$$

where

$$\bar{\sigma} = \begin{bmatrix} \bar{\sigma}_0 & & & & \\ & \sigma_1 & & & \\ & & \sigma_2 & & \\ & & & \ddots & \\ & 0 & & & \sigma_{n_2} \end{bmatrix} \quad (30)$$

The alias matrix reveals much of the probability structure of the estimation procedure. Equations 26, 28, 29, and 30 show that the standard deviation of the i^{th} adjusted gravity anomaly is the root sum square (RSS) of the terms in the i^{th} row of the alias matrix. The elements in the first column of the alias matrix represent the RSS contribution to the standard deviation of each estimated parameter caused by the data noise. The elements in the J^{th} column, $J \geq 2$, represent the RSS contribution to the standard deviation of each estimated parameter attributable to the $J - 1^{st}$ unadjusted parameter. These terms are called *the aliasing contributions* to the uncertainty in the adjusted parameters attributable to the uncertainty in the $J - 1^{st}$ unadjusted parameter. Note that the aliasing contributions attributable to the J^{th} unadjusted parameter are proportional to the standard deviation of the J^{th} parameter. Definition: In a given estimation process, the i^{th} estimated parameter is said to be *orthogonal* with respect to the J^{th} unestimated parameter if the aliasing contribution to the i^{th} estimated parameter attributable to the uncertainty of the J^{th} unestimated parameter is zero.

Two things can be noted about this definition. First, the orthogonality relationship between two parameters must be defined within the context of a given estimation process. The data set must be defined, and it must be stipulated which parameters are in the adjusted and the unadjusted modes. Second, although the discussion has been of mean free-air gravity anomalies and altimeter data, the mathematical results and definitions are applicable to any linear estimation problem in which some parameters are adjusted and others are left unadjusted.

To see the implications of the orthogonality relationship, a more revealing representation of the aliasing terms is necessary. First, note that the first term on the right side of equation 22 is the covariance matrix of the estimation process under the assumption that the unadjusted parameters are perfectly known. This covariance matrix gives the uncertainty of the estimates attributable to data noise only. Define the so-called “noise-only” covariance matrix as

$$\bar{P} = (A_1^T Q^{-1} A_1 + P_1^{-1})^{-1} \quad (31)$$

Next, observe that the elements in the i^{th} row and J^{th} column of A_1 and A_2 , respectively, are the partial derivative of the i^{th} data point with respect to the J^{th} adjusted parameter and the partial derivative of the i^{th} data point with respect to the J^{th} unadjusted parameter. The aliasing contribution to the i^{th} adjusted parameter attributable to the J^{th} unadjusted parameter can be written as

$$L(I, J + 1) = \sum_{K=1}^{n_1} \bar{P}(I, K) \sum_{L=1}^m \frac{\partial \delta \tilde{N}(L)}{\partial \delta \tilde{g}_1(K)} Q^{-1}(L, L) \frac{\partial \delta \tilde{N}(L)}{\partial \delta \tilde{g}_2(J)} \quad (32)$$

where m is the number of data points.

If the estimates of the adjusted parameters are relatively uncorrelated in the noise-only covariance matrix, equation 32 can be approximated by

$$L(I, J + 1) = \bar{P}(I, I) \sum_{L=1}^m \frac{\partial \delta \tilde{N}(L)}{\partial \delta \tilde{g}_1(I)} Q^{-1}(L, L) \frac{\partial \delta \tilde{N}(L)}{\partial \delta \tilde{g}_2(J)} \quad (33)$$

A sufficient condition for the left side of equation 21 to approximate zero is for the observability patterns of $\delta \tilde{g}_1(I)$ and $\delta \tilde{g}_2(J)$ in the data to be virtually nonoverlapping. Up to about 40 degrees, Stokes' function rapidly attenuates with increasing values of spherical radius (Reference 4). Hence, if the grids on which $\delta \tilde{g}_1(I)$ and $\delta \tilde{g}_2(J)$ are defined are sufficiently separated, the orthogonality relationship will be effectively satisfied and the estimate of $\delta \tilde{g}_1(I)$ will not exhibit aliasing from the uncertainty of $\delta \tilde{g}_2(J)$. Conversely, if the grid on which $\delta \tilde{g}_1(I)$ was defined were in close proximity to grids whose gravity anomalies were unadjusted, serious aliasing of the resultant estimate would be expected.

It should be clear then, that if the gravity anomalies are estimated in a block, the outer layers of the block contain gravity anomalies whose estimates will be badly aliased by the adjacent unadjusted parameters, and must therefore be discarded. However, the gravity anomalies in a sufficiently small inner core of the block may be adequately separated from the unadjusted parameters to be effectively orthogonal with respect to them. The estimates of these terms will presumably be of sufficient accuracy that they can be accepted. In effect, for every block of gravity anomalies that will be estimated, it will be necessary to construct a "buffer zone" several layers deep of gravity anomalies which surround the block. The new and larger block of gravity anomalies must be simultaneously estimated, and the estimates of gravity anomalies in the buffer zone must then be rejected because of aliasing. To achieve an intelligent compromise between estimation accuracy and computational load, it is necessary to determine the relationship between the depth of the buffer zone and the accuracy of the estimation procedure. The relationship will vary with grid size, computation radius, and data density. The most convenient way to study the relationship is to use covariance analysis

techniques to generate alias matrices for several situations and to attempt generalizations from the results. This is done in the following section.

ESTIMATION STRATEGIES FOR GRAVITY ANOMALY DETERMINATION

The accuracy with which a given gravity anomaly is estimated from altimeter data is a function of many parameters. It is, of course, dependent on the accuracy and density of altimeter data. As explained in the previous section, it is also dependent on the radial distance between the estimated gravity anomaly and the nearest gravity anomaly which is in the unadjusted mode. This parameter is called the estimation radius. Another parameter, the computation radius, is an important determinant of the accuracy of a gravity anomaly estimation. The computation radius sets the maximum distance from a given grid element over which data is to be processed in order to estimate the gravity anomaly defined on that grid element. Estimation accuracy does not necessarily increase with increasing computation radius. Note that, for a given estimation radius, the covariance matrix of a set of estimated gravity anomalies is given by equation 22 as the sum of a matrix which is dependent only on data uncertainty and a matrix which represents the aliasing effects from the unadjusted parameters. With increasing computation radius, the elements of the first matrix must decrease, but, in general, the elements of the matrix which conveys the aliasing effects will increase. This effect can be shown graphically by means of so-called aliasing maps. The aliasing maps of figures 3 and 4 were made simultaneously in a covariance mode, a situation in which the marine geoid was described by means of 2- by 2-degree gravity anomalies and by assuming 12 altimeter observations for each grid, each with an uncertainty of 1 meter. In figure 3, the RSS contribution was mapped in milligalileos (mgal)* to the uncertainty in the estimated gravity anomaly defined on the grid element in the lower left-hand corner when the adjacent anomalies were assumed to be in an unadjusted mode and to have an *a priori* uncertainty of 0.50 millimeters/sec² (mm/s²) (50 mgal). The computation radius was 5 degrees. Note that the aliasing contributions decrease with the distance between the unadjusted parameter and the estimated parameter, demonstrating the inherent orthogonality property of the gravity anomaly parameterization of the marine geoid. In figure 4, the computation radius has been changed to 20 degrees, and the aliasing effect decreased much less radically.

To achieve an intelligent compromise between computational load and estimation accuracy, it is necessary to determine the precise relationship between the accuracy of the estimate of a given gravity anomaly and the estimation radius and computation radius employed in the estimation procedure. To do this, it was assumed that altimeter data existed at a density of three data points per 1- by 1-degree grid and that the uncertainty on the data was 1 meter. It was also assumed that unadjusted gravity anomalies had a standard error about zero of 0.50 mm/s² (50 mgal). This figure is conservative; a more realistic figure is 0.30 mm/s² (30 mgal) (Reference 12). No *a priori* estimate was assumed for estimated parameters. The effect of unadjusted parameters out to a spherical radius of 45 degrees were included in the

*Throughout the text of this document, the measurement unit of milligalileo (mgal) has been converted to a Standard International Unit of millimeters/sec² (mm/s²). For convenience, the illustrations have not been converted. The simple conversion is 1 mgal = 0.01 mm/s².

1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	0	0
2	2	2	2	2	2	2	1	1	1	1	1	0	0	0
3	2	2	2	2	2	2	2	2	1	1	1	1	0	0
3	3	3	3	3	2	2	2	2	2	1	1	1	1	0
4	3	3	3	3	3	3	2	2	1	1	1	1	1	1
5	4	4	4	4	3	3	3	2	2	2	1	1	1	1
5	5	4	4	4	4	4	3	2	2	2	2	1	1	1
6	6	6	5	5	5	4	3	3	2	2	2	1	1	1
8	8	7	6	5	5	5	3	3	3	2	2	2	1	1
10	10	9	8	6	5	5	4	3	3	2	2	2	1	1
18	19	14	9	7	6	6	4	4	3	3	2	2	1	1
31	26	19	10	8	6	6	5	4	3	3	2	2	2	1
	31	18	10	8	6	5	4	3	3	2	2	2	1	1

Figure 3. Alias map for 2- by 2-degree grids and for 5-degree computation radius.

4	4	4	4	4	4	4	3	3	3	3	2	2	2	1
4	4	5	5	5	5	4	4	4	3	3	3	2	2	2
5	6	6	6	6	5	5	5	5	4	4	3	3	2	2
6	7	7	7	7	7	6	6	5	5	4	4	3	3	2
8	8	9	9	8	8	8	7	7	6	5	4	4	3	3
10	10	11	11	11	10	10	9	8	7	6	5	4	4	3
12	14	14	14	14	13	12	11	10	8	7	6	5	4	3
15	16	17	17	16	15	14	13	11	9	7	6	5	4	4
17	19	19	20	19	17	16	14	12	10	8	7	6	5	4
19	22	22	22	21	19	17	16	13	10	9	7	6	5	4
22	24	25	24	22	21	19	16	14	11	9	7	6	5	4
25	28	28	26	24	22	19	17	14	11	9	7	6	5	4
28	32	30	28	25	22	20	17	14	11	9	7	6	5	4
36	34	32	28	24	22	19	16	14	11	9	7	6	5	4
	36	28	25	22	19	19	15	13	10	8	7	6	5	4

Figure 4. Alias map in mgals for 2- by 2-degree grids and for 20-degree computation radius.

computations. Under these conditions, figure 5 provides the standard deviation in the estimates of 5- by 5-degree mean free-air gravity anomalies as a function of estimation radius and computation radius. This figure shows that the most efficient estimation strategy for 5- by 5-degree anomalies is obtained with an estimation radius of about 10 degrees and a computation radius of about 5 degrees. This strategy will result in an accuracy of 0.01 mm/s^2 (1 mgal). This implies that if 5- by 5-degree mean free-air gravity anomalies are estimated in blocks, estimates of gravity anomalies in the outer two layers of the block should be discarded because of aliasing. The rest of the estimated gravity anomalies should be accurate to within 0.01 mm/s^2 (1 mgal), provided a 5-degree computation radius is used. It is important to remember, however, that the prime intent is to estimate a marine geoid rather than gravity anomalies. With estimates of 5- by 5-degree mean free-air gravity anomalies, Stokes' formula can be used to analytically reconstruct a marine geoid which is sufficiently detailed that 5-degree features can be noted. Because the discrete form of Stokes' formula is linear, it is easy to propagate a given error in gravity-anomaly estimation into an error in marine-geoid determination. Assuming that Stokes' formula is accurate if its summation is carried out within a 90-degree spherical radius of a given point, an 0.01-mm/s^2 (1-mgal) standard deviation in the estimate of 5- by 5-degree mean free-air gravity anomalies propagates into a 40-cm standard deviation of the resultant marine geoid. Therefore, by applying proper estimation procedures to the reduction of altimeter data, determination of 5-degree features of the marine geoid with a resolution of 40 cm should be possible.

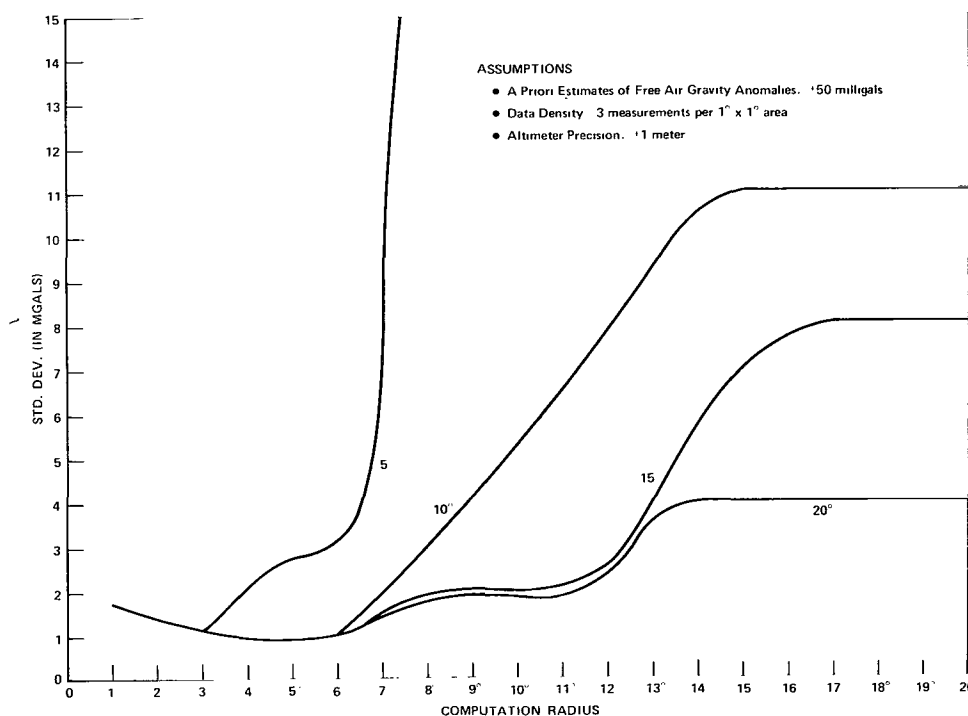


Figure 5. Accuracy of 5- by 5-degree mean free-air gravity anomaly estimate versus computation radius for various estimation radii.

To obtain a more detailed marine geoid presents a more difficult estimation problem. Figure 6 provides the standard deviation in the estimates of 3- by 3-degree mean free-air gravity anomalies as a function of estimation radius and computation radius. An estimation radius of 12 degrees and a computation radius between 10 and 11 degrees appears to provide the most intelligent estimation strategy. The strategy leads to estimates of 3- by 3-degree mean free-air gravity anomalies with standard deviations of 0.05 mm/s^2 (5 mgal). This implies that, if 3-degree mean free-air gravity anomalies are estimated in blocks, the outer four layers must be discarded as being badly aliased. An 0.05-mm/s^2 (5-mgal) standard deviation in estimates of 3- by 3-degree mean free-air gravity anomalies propagates into a standard deviation of 1.2 m in the resultant marine geoid. With the present assumptions, no estimation strategy is adequate to determine features of the marine geoid as small as 2 degrees. With much greater data densities, such fine resolution is possible. For instance, if we increase the data density by a factor of 100, thus postulating 300 data points for each 1- by 1-degree block, then, with an estimation radius of 10 degrees and a computation radius of 2 degrees, 2- by 2-degree anomalies can be estimated with a standard deviation of 0.023 mm/s^2 (2.3 mgal). This propagates into a standard deviation of the marine geoid of about 42 cm. However, it is probably not realistic to postulate such data densities, and, in addition, it suggests a very heavy computational load. The accurate resolution of 3-degree features of the marine geoid is likely to be the limit of what can be accomplished with altimeter data and the Stokes' formula mean free-air gravity anomaly parameterization of the marine geoid.

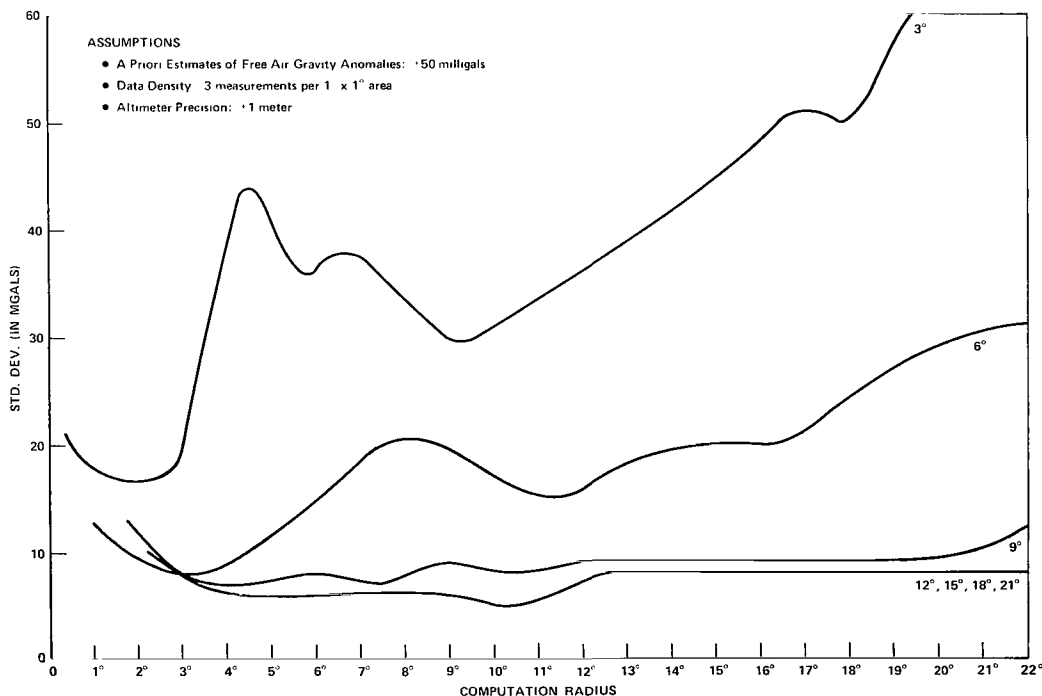


Figure 6. Accuracy of 3- by 3-degree mean free-air gravity anomaly estimate versus computation radius for various estimation radii.

Finally, note that these results were accomplished without assuming *a priori* estimates of gravity anomalies. In fact, these estimates have been derived from satellite tracking data and direct gravity measurements. When these estimates are optimally combined with altimeter data by means of equation 10, the resultant estimates of gravity anomalies will be predicted on all relevant data. The representation of the marine geoid derived from these estimates will then constitute the best possible estimate of that surface.

SUMMARY

Data from the Skylab altimeter have demonstrated the ability of altimetry to discern the fine structure of the marine geoid. However, during the GEOS-C mission, global sensing of sea-surface topography from a spacecraft-borne altimeter will be possible for the first time. To make optimal use of the vast amount of altimeter data expected from the GEOS-C, numerous corrections must be made to the data, both to eliminate systematic errors inherent in the data type itself and to correct for the effects of deviations of the mean sea surface from the marine geoid.

To employ standard parameter-estimation techniques for estimating a marine geoid from altimeter data, it is necessary to use a parameterization of the marine geoid which exhibits good orthogonality characteristics in the data type. In this respect at least, the Stokes' formula mean free-air gravity anomaly parameterization appears to be the most promising. For a given grid size and data density, the accuracy of the geoid resulting from an estimate of gravity anomalies from altimeter data is a strong function of the choice of estimation radius and computation radius. An application of covariance analysis techniques reveals that, assuming a data density of three measurements per 1- by 1-degree spherical surface area with each measurement accurate to within 1 meter, optimal choices of estimation radius and computation radius of 10 and 5 degrees, respectively, yield an accuracy in the estimate of 5- by 5-degree mean free-air gravity anomalies of approximately 0.01 mm/s^2 (1 mgal). This result implies that, with a judicious choice of estimation strategy, geoid height can be determined with a standard deviation of 40 cm and a spatial resolution of 500 km (5 arc degrees). With the same data density and data accuracy assumptions, covariance analysis demonstrates that optimal choices of 12 degrees for estimation radius and 10 degrees for computation radius permits one to estimate with an accuracy of approximately 1.2 m in geoid height with a spatial resolution of 300 km (3 arc degrees). Severe aliasing difficulties are encountered in attempting to estimate more detailed geoids. Our covariance analysis studies indicate, however, that a very small computation radius, together with great data densities, can mitigate the aliasing effects and yield in localized areas a marine geoid capable of showing spatial resolutions at the 100- to 200-km level.

Finally, note that use of the Stokes' formula mean free-air gravity anomaly parameterization of the marine geoid makes it easy to combine satellite tracking data and direct measurements of gravity anomalies with altimeter data to obtain an optimal estimate of the marine geoid.

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